Relativistic Mean Field Models of Neutron Stars

Ilona Bednarek

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Chapter 1

Introduction

The physical properties of matter under extreme conditions of temperature and density have received special attention during the past years and this is still one of the most complex problems of theoretical and experimental physics [1, 2, 3, 4, 5]. The state of matter under a given set of physical conditions can be described by a constitutive relation known as an equation of state (EOS). It provides a mathematical relation between state functions associated with matter and is a key factor in many essential aspects of stellar physics such as the ionization and excitation states, the onset of electron degeneracy or the temperature gradient [6, 7, 8].

The EOS together with nuclear reaction rates and the opacity determines the structure and evolution of a star. The longest phase of stellar evolution described by models of stars in thermal and hydrostatic equilibrium with central hydrogen burning are characterized by the EOS of a classical perfect gas. As a star evolves and its density increases, electrons which initially form a perfect, classical gas start to form a dense, degenerate quantum gas. The EOS of such a degenerate gas is an increasing function of density. The form of this function changes when the electron gas is considered in a non-relativistic or ultra-relativistic regime. In the case of a white dwarf, this is the pressure of a degenerate electron gas that supports the star [9].

Having obtained the EOS of degenerate electron gas and scaling it trivially with particle masses and statistical weight, the EOS adequate for the degenerate perfect gas of arbitrary fermions can be obtained. This is of special relevance for neutrons as they are the main component of a neutron star. In general, neutron star models are constructed at different levels of complexity starting with the most elementary, this assumes that neutrons are the only component of neutron star matter. Realistic calculations of the properties of neutron stars are based upon the relativistic equation for hydrostatic equilibrium and an EOS for neutron star matter. Its computation is the main problem in the construction of a reliable model of a neutron star.

The aim of this dissertation is to analyze the properties of neutron star matter in high density and neutron-rich regimes and to study neutron star parameters, namely the masses and radii which are the most sensitive for the form of the EOS.

The description of a neutron star interior is modelled on the basis of the EOS of a dense nuclear system in a neutron-rich environment [9, 10]. Despite the fact that neutron star matter is directly affected by the nature of strong interactions, it is not possible to give its description on the basis of quantum chromodynamics (QCD) even though it is the fundamental theory of strong interactions. At the hadronic energy scale where the experimentally observed degrees of freedom are hadrons, the direct description of nuclei in terms of QCD becomes inadequate. Another alternative approach had to be formulated. In general the description of nuclear matter is based on different models which can be grouped into phenomenological and microscopic. Additionally each one of them can be either relativistic or non-relativistic.

In a microscopic approach the construction of the realistic model of nucleon-nucleon (NN) interaction can be inspired by the meson exchange theory of nuclear forces. The parameters within the model have to be adjusted to reproduce the experimental data for the deuteron properties and NN scattering phase shifts [11]. Defining the nuclear Hamiltonian is a starting point in the description of the nuclear matter. The next step requires solution of the many-body problem.

The basic approaches to this are of variational-type [12], [13] and of Brueckner-type. The solution of the nuclear many-body problem performed with the use of variational calculations for realistic NN interactions (for example for the Argonne v14 or Urbana v14 potentials) saturate at the density $\sim 2 \times \rho_0$, where ρ_0 denotes the saturation density [12], [13]. In order to obtain the correct description of nuclear matter properties, namely the saturation density, binding energy and compression modulus at the empirical values, a phenomenological three-nucleon interaction has to be introduced. Two-body forces, together with implemented three-body forces, help providing the correct saturation point of symmetric nuclear matter [14, 15]. The nuclear matter EOS calculated with the use of the Brueckner–Hartree–Fock [16, 17] approximation with the employed realistic two-nucleon interactions (the Bonn and Paris potentials) also does not correctly reproduce nuclear matter properties. Thus there are attempts to consider the nuclear interaction problem in a relativistic formalism. The relativistic version of the Brueckner–Hartree–Fock approximation – the Dirac–Brueckner–Hartree–Fock approach [18, 19, 20, 21] is also based on realistic NN interactions. A qualitative description of the strong short-range NN force with large repulsive components compensated by attractive ones can be provided within the meson exchange model for the NN interaction. In its simplest version the meson exchange model involves only two types of mesons – the vector meson ω and the scalar meson σ. The nucleon self-energy involved in the Dirac equation is calculated with the use of the meson exchange model within a Hartree approximation.

The phenomenological model of the EOS can be calculated on the basis of the density dependent effective NN interactions. Calculations performed in Hartree–Fock [22] and Thomas–Fermi [23] approximations for the most popular phenomenological NN forces – the Skyrme forces, which contain parameters that have been established by adjusting nuclear matter and finite nuclei properties, yields very good results. Another phenomenological approach for the nuclear many-body problem is based on relativistic field theory. This relativistic approach to nuclear matter at the hadronic energy scale was developed by Walecka. The formulated theory known as quantum hadrodynamics (QHD) [24, 25, 26, 27, 28, 29, 30] gives a quantitative description of the nuclear many-body problem [25, 31]. In this model the force between nucleons is thought of as being mediated by the exchange of mesons. The original model (QHD-I) contains nucleons interacting through the exchange of simulating medium range attraction σ mesons and ω mesons responsible for the short range repulsion. The extension of this model (QHD-II) [32, 33, 34, 35] also includes the isovector meson ρ. Nonlinear terms of the scalar and vector fields were added in order to get the correct value of the compressibility of nuclear matter and the proper density dependence in vector self-energy.

Furnstahl, Serot and Tang [36, 37], using Lorentz-covariant effective quantum field theory and density functional theory (DFT) for hadrons, have constructed an effective Lagrangian consistent with the underlying symmetries of QCD. This Lagrangian includes nucleons, pions and non-Goldstone bosons which provide the description of the medium and short range of the NN interaction and the nonvanishing expectation values of bilinear nucleon operators of the nucleon fields: $\bar{N}N$, $\bar{N}\gamma^{\mu}N$. Due to the nonlinear realization of the chiral symmetry, there is the possibility of introducing a light scalar, isoscalar, chiral singlet field with the Yukawa coupling to nucleons. This scalar field simulates the effect of pion exchange and describes the attractive NN interaction. Applying Georgi and Manohar's naive dimensional analysis and naturalness [38, 39, 40] the nonlinear chiral Lagrangian can be expanded in powers of the fields and their derivatives. The assignment of an index ν to each term which appears in the effective Lagrangian makes it possible to truncate this Lagrangian to terms with $\nu \leq 4$. Owing to a very high density of matter in neutron star interiors, extrapolation of nuclear models for such dense systems is required for its description. Contrary to satisfactory results obtained for finite nuclei at saturation density the predictions made by nuclear models for much more dense matter differ considerably from each other. Calculations performed on the basis of Dirac–Brueckner–Hartree–Fock model lead to a rather soft EOS. Moreover, recent experimental data has suggested a similar conclusion. However, the standard nonlinear models of quantum hydrodynamics give EOS which is too stiff for increasing density.

The TM1 parameter set constructed by Sugahara and Toki [31] includes the quartic vector self-interaction term gives satisfactory results for finite nuclei, neutron star matter and supernova models. It improves the results of the NL1 [28] and NL3 [41] nonlinear models. The parameter set G2 which stems from effective field theory is the original parameter set of Furnstahl, Serot and Tang. It gives the EOS where high density behavior resembles the DBHF result [42, 43]. Del Estal et al. [44, 45] have constructed a parameter set which at saturation gives the same nuclear matter properties as the TM1 parameter set. However, the behavior of the EOS calculated for these parameters, due to the presence of additional nonlinear couplings, at densities above the saturation density is similar to the one calculated for the G2 parameter set. Usage of the TM1* and G2 parameter sets for the description of asymmetric neutron star matter is intended to make this description more detailed and complete. For comparison similar calculations have been done for neutron star matter with TM1 parameterization supplemented by the inclusion of nonlinear couplings [46, 47]. The analysis has been carried out by adding the nonlinear mixed isoscalar-isovector meson interaction terms to the TM1 parameter set. This enlargement, due to the coupling to the isovector meson field influences the density dependence of the symmetry energy and affects the chemical composition of neutron stars changing the proton and lepton profiles in the case of a cold neutron star and additionally the cooling rate and neutrino flux of a proto-neutron star.

As stated above the description of a neutron star requires taking into consideration not only its interior region but also its remaining parts, namely the inner and outer crust and surface layers. The composite EOS, which allows us to calculate neutron star structure for the entire density span, can be constructed by adding the Baym–Pethick–Sutherland [48] EOS, for very low densities $n < 0.001\,\mathrm{fm}^{-3}$, and for the Machleidt–Holinde–Elster and Bonn [49] and Negele–Vautherin [50] forms of the EOS densities within the range of $0.001\,\mathrm{fm}^{-3} < n < 0.08\,\mathrm{fm}^{-3}$.

Normal nuclei bound by strong forces are in states, which can be defined as the equilibrium state of isospin symmetric nuclear matter with minimum energy per nucleon. The condition of an equal number of protons and neutrons is fulfilled in the case of infinite symmetric nuclear matter. In heavy nuclei the overall neutron excess defined as $I = (N - Z)/(N + Z)$ takes the value up to about 0.24 [51]. This limit can be exceeded in rare-isotope accelerator experiments. Neutron star matter realizes the condition of extreme asymmetry which can approach the value of 0.95. It is gravity that binds neutron stars in this case. Highly asymmetric neutron star matter is not bound by the strong interaction.

The high asymmetry of neutron star matter implies the presence of leptons. The indispensable conditions of charge neutrality and chemical equilibrium constrain the neutron star matter EOS determining the main differences between the neutron star matter EOS and the one relevant to infinite symmetric nuclear matter.

In the interior of neutron stars the density of matter could exceed normal nuclear matter density by a factor of few. In such high density regimes nucleon Fermi energies exceed the value of hyperon masses and thus new hadronic degrees of freedom are expected to emerge. The higher the density the more various hadronic species are expected to populate. The presence of hyperons adds another very important aspect to the problem of the nuclear EOS. The strange nuclear matter created during the heavy ion collision experiments can be observed over such a short period of time that the weak decays of strange baryons can be neglected and the system is characterized by zero net strangeness. In contrast, strangeness is not conserved in neutron star matter. This has very important consequences for its EOS and also has a direct effect on neutron star parameters. The onset of hyperon formation depends on the hyperon-nucleon and hyperon-hyperon interactions. Hyperons can be formed both in leptonic and baryonic processes. Several relevant strong interaction processes proceed and establish the hadron population in neutron star matter.

In this dissertation all calculations have been done with the use of relativistic mean-field approximation. A very important aspect of neutron star observations is connected with the fact that measurements of neutron star parameters can provide constraints on the form of the EOS and thereby improves our understanding of matter at extreme pressure and density.

This dissertation is organized in the following way: it starts from the chapters which present the stellar structure equations and thermodynamic properties of stellar matter. In chapter 4 the ground-state nuclear matter properties are described. This has been done on the basis of a nuclear matter EOS which is specified by the following parameters: the binding energy, density at saturation, incompressibility, symmetry energy coefficient, the slope and curvature of the symmetry energy. In chapter 6 the relativistic mean field models, which are based on quantum hadrodynamics and which have been successfully applied to the description of nuclear matter properties are presented. Chapter 7 describes in outline the main findings of the Furnstahlm, Serot and Tang model known as effective relativistic mean field theory which includes new, general couplings. In this chapter the extension of the considered models has been examined by including strangeness. The nuclear matter properties have been calculated for the chosen parameter sets, namely TM1, G2 and TM1*. In the case when the additional nonlinear couplings and δ meson were taken into account the parameters in the isovector channel have been determined. Special efforts have been made to produce an optimal set of parameters for the strange sector of each model. Chapter 8 presents the results of the numerical analysis of nuclear matter properties for the selected parameter sets. Subsequent sections of this chapter give the results for infinite symmetric nuclear matter, symmetric nuclear matter with nonzero strangeness, asymmetric nuclear matter and asymmetric strangeness-rich matter.

Special attention has been devoted to neutron star matter which exemplifies infinite, asymmetric nuclear matter in β-equilibrium. After discussing the equilibrium conditions for neutron star matter in chapter 9, the EOS has been constructed for the selected parameterizations. On obtaining the EOS the composition and structure of a neutron star have been analyzed. The obtained mass-radius relations are also constructed for neutron star matter with hyperons and compared with the mass-radius relation for non-strange matter. This has been done in chapter 10. The analysis has been performed for the ordinary TM1 parameter set enlarged by the additional nonlinear meson interaction terms and for the parameter sets of the models which have been constructed on the basis of the relativistic effective field theory. The TM1 parameter set gives larger neutron star masses than the TM1* and G2. However, the key difference between the TM1 and TM1* and G2 mass-radius diagrams lies in the results obtained for the strong hyperon-hyperon interaction. In this case for TM1* and G2 parameter sets besides the ordinary stable neutron star branch there exists the additional stable branch of solutions which are characterized by a similar value of masses but with significantly reduced radii. Chapter 11 presents the astrophysical constraints on the EOS. In chapter 12 proto-neutron star models calculated on the basis of the presented parameter sets have been analyzed. Finally the findings of all performed calculations and analysis have been summarized.

Ilona Bednarek

Modele gwiazd neutronowych w ujęciu relatywistycznej teorii pola średniego

Streszczenie

Celem pracy było zbadanie własności asymetrycznej materii jądrowej, ze szczególnym uwzględnieniem obszaru wysokich gęstości charakterystycznych dla rdzeni gwiazd neutronowych. Przeprowadzona analiza obejmowała nieliniowe modele Walecki, z dodatkowymi członami nieliniowymi uwzględniającymi oddziaływanie mezonu skalarnego σ i mezonu wektorowego ρ oraz mezonów wektorowych ω i ρ, jak również modele skonstruowane na podstawie efektywnej teorii pola – model Furnstahla, Serota i Tanga (FST). Obliczenia zostały wykonane w przybliżeniu relatywistycznej teorii pola średniego. Dodatkowe rozszerzenie sektora izoskalarnego obejmowało wprowadzenie mezonu izowektorowego δ. W obszarze bardzo wysokich gęstości energie Fermiego neutronów przyjmują tak duże wartości, że korzystna energetycznie staje się ich konwersja na hiperony. Uwzględnienie dziwności w rozpatrywanych modelach materii jądrowej pozwoliło na szerszą analizę dodatkowych klas modeli, w których został zbadany wpływ oddziaływania nukleon–hiperon i hiperon–hiperon na postać równania stanu materii gwiazdy neutronowej.

Równanie stanu jest najważniejszym czynnikiem decydującym zarówno o strukturze, jak i o parametrach gwiazdy neutronowej.

Szczególnie interesujące są wartości mas maksymalnych obliczone dla poszczególnych modeli. W przypadku modeli uzyskanych na gruncie efektywnej teorii pola i następnie rozszerzonych przez uwzględnienie niezerowej dziwności rozwiązania równania Oppenheimera–Volkoffa–Tolmana wskazują na istnienie trzeciej rodziny stabilnych, zwartych gwiazd o gęstościach przekraczających gęstości występujące w rdzeniach gwiazd neutronowych.

Ilona Bednarek

Relativistische Mittelfeld-Modelle von Neutronensterne

Zusammenfassung

Ziel der Arbeit war, die assymetrische Eigenschaft der Kernmaterie unter besonderer Berücksichtigung des Bereichs hoher Dichten, die für Kerne der Neutronensternen charakteristisch sind, zu erforschen. Die durchgeführte Analyse umfasste nicht lineare Walecki-Modelle mit zusätzlichen, nicht linearen Segmenten, welche die Wechselwirkung des Skala-Mesons σ und des Vektor-Mesons ρ, der Vektor-Mesonen ω und ρ, sowie Modelle, die in Anlehnung und die effektive Feldtheorie – das Modell von Furnstahl, Serot und Tang (FST) konstruiert wurden, berücksichtigen. Die Berechnungen wurden in Annäherung an die relativistische Mittelfeldtheorie durchgeführt. Zusätzliche Erweiterung des isoskalaren Sektors umfasste die Einführung des Isovektores δ. Im Bereich sehr hoher Dichten nehmen die Fermie – Energieen von Neutronen so grosse Werte ein, dass energetisch günstig ihre konversion in Hyperonen wird. Die Berücksichtigung der Strangeness in den untersuchten Modellen der Kernmaterie liess eine breitere Analyse zusätzlicher Modellklassen vornehmen, bei denen der Einfluss der Wechselwirkungen Nukleon – Hyperon und Hyperon – Hyperon auf die Form der Zustandsgleichung der Materie eines Neutronensternes untersucht wurde. Die Zustandsgleichung der Materie eines Neutronensternes untersucht wurde.

Die Zustandsgleichung ist der wichtigste Faktor, der sowohl über die Parameter eines Neutronensternes entscheidet.

Besonders interessant sind die maximalen Massenwerte für die einzelne Modelle berechnet. Im Fall von Modellen, die aufgrund der Effektiven Feldtheorie erreicht und anschliessend durch Berücksichtigung der Nichtnull – Strangeness erweitert wurden deutet die Lösung der Oppenheimer – Volkoff – Tolman Gleichung auf die Existenz einer dritten Familie stabiler, einheitlichen Sterne hin, deren Dichten die in den Kernen von Neutronensternen auftretenden Dichten beschreiten.